II-7 ELECTRICALLY SHORT 90° COUPLERS UTILIZING LUMPED CAPACITORS

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The ultimate size of many microwave components such as beam-forming networks, switching trees, amplifiers, mixers, comb filters, etc., is related to the size of a 90° coupler. The cross-section of a coupler can be reduced by means of todays integrated circuit technology. This paper presents a method of reducing the length of a coupler over and above length reductions possible by means of the utilization of a high dielectric constant medium.

An electrically short 90° coupler could be constructed if it were possible to implement the lumped element equivalent of a 90° hybrid. The ultimate size of the couplers would then be limited only by the size of the lumped capacitances and inductances. The lumped element equivalent circuit of a 90° hybrid is shown in Figure 1. Oliver has derived the equations for match, isolation and coupling in terms of L, the self-inductance per unit length: M, the mutual inductance per unit length; C_m, the mutual capacitance per unit length; and C, the self-capacitance per unit length.

The construction of the lumped capacitances (mutual and self) presents no serious problems with the present state-of-the-art, but the construction of small lumped self and mutual inductances is a formidable task at microwave frequencies. In this paper we discuss means of constructing electrically short 90° coupler utilizing lumped capacitors. The equivalent of the lumped inductances will be obtained by means of a high impedance length of coupled transmission line.

A short length of high impedance transmission line can be approximated by the same length of lower impedance line plus a fixed series inductance. In a similar manner, a short length of high impedance coupler (a coupler which would match at a high impedance level) can be approximated by a short length of a lower impedance coupler and lumped inductances as shown in Figure 2:
The addition of lumped capacitors (self and mutual) can convert the excess lumped inductances of Figure 2 into a portion of a lumped element hybrid.

A high impedance coupler with periodic capacitive loading is shown in Figure 3. The sections of high impedance coupling are equivalent to a Y_o coupler (a coupler that would be matched with a Y_o characteristic admittance) with lumped inductances (Figure 2). If the lumped reactances are adjusted properly they will act as a small section of a Y_o coupler. The periodic coupler of Figure 3 would then be equivalent to a Y_o coupler of a longer length.

The above explanation is purely heuristic. Let us look at a cell of the periodic coupler of Figure 3. If the cell can be matched, the whole coupler will be matched (match is sufficient for a coupler). The eigenvalues of the scattering matrix (Y_o characteristic admittance) of the cell are:

$$\Gamma^{oo} = - e^{-j\,2\,\mathrm{ctn}^{-1} B^{oo}}$$

$$\Gamma^{eo} = - e^{-j\,2\,\mathrm{ctn}^{-1} B^{eo}}$$

$$\Gamma^{oe} = - e^{-j\,2\,\tan^{-1} B^{oe}} \qquad (1)$$

$$\Gamma^{ee} = - e^{-j\,2\,\tan^{-1} B^{ee}}$$

where

$$B^{oo} = \frac{1}{KY_1}\left[Y_{1o}\,\mathrm{ctn}\,\frac{\beta\ell}{2} - B_m - B\right]$$

$$B^{eo} = \frac{1}{KY_1}\left[Y_{1e}\,\mathrm{ctn}\,\frac{\beta\ell}{2} - B\right]$$

$$B^{oe} = \frac{1}{KY_1}\left[Y_{1o}\tan\frac{\beta\ell}{2} + B_m + B\right]$$

$$B^{ee} = \frac{1}{KY_1}\left[Y_{1e}\tan\frac{\beta\ell}{2} + B\right]$$

where

$B = \omega C$

$B_m = 2\,\omega\,C_m$

Y_{1o} = odd mode admittance

Y_{1e} = even mode admittance

$Y_1 = \sqrt{Y_{1o}\;Y_{1e}}$

$K = \dfrac{Y_o}{Y_1}$

In order for the cell to be matched, equations (2) and (3) must be satisfied simultaneously:

$$\Gamma^{oo} = -\Gamma^{ee}$$

or (2)

$$B^{oo}\,B^{ee} = 1$$

$$F^{oe} = -\Gamma^{eo}$$

or (3)

$$B^{oe}\,B^{eo} = 1$$

Combining equations (2) and (3) we obtain:

$$\frac{Y_{1o}}{Y_{1e}} = \frac{B_m + B}{B} \tag{4}$$

Hence, equations (2) and (4) are necessary for a match. Equation (4) dictates that the ratio of lumped mutual capacitance to lumped self-capacitance is determined by the coupling coefficient of the high impedance coupling section. Also note that equation (4) is frequency independent. Equation (2) can be expanded as:

$$1 - \left(\frac{B_m}{Y_1} + \frac{B}{Y_1}\right)\left(\frac{B}{Y_1} + \frac{Y_{1e}}{Y_1}\tan\frac{\beta\ell}{2}\right) + \frac{B\,Y_{1o}}{Y_1^{\,2}}\,\mathrm{ctn}\,\frac{\beta\ell}{2} = K^2 \tag{2}$$

For small $\beta\ell$ this equation reduced to:

$$\frac{B\,Y_{1o}}{Y_1^{\,2}}\,\mathrm{ctn}\,\frac{\beta\ell}{2} = K^2 - 1 \tag{5}$$

where

$$Y_1 = \sqrt{Y_{1o}\,Y_{1e}}$$

This equation is essentially frequency independent since B and ctn $\frac{\beta\ell}{2}$ have equal and opposite frequency slopes for small $\beta\ell$

When equations (4) and (5) are simultaneously met for small $\beta\ell$ the resulting coupler has a coupling parameter ρ where:

$$\rho = \frac{Y_{oo}}{Y_{oe}} = \frac{Y_{1o}}{Y_{1e}}$$

The length of the periodically loaded coupler is shorter than the distributed coupler by a factor of K ($K = \frac{Y_o}{Y_1}$)

Since the size reduction is proportional to Y_o ($Y_o = \frac{1}{Z_o}$) this type of coupler will function best in a low impedance system.

The number of cells required will depend upon the desired bandwidth. A single cell will provide nearly an octave of bandwidth. The required values of K and $\frac{2B}{Y_1}$ are plotted as a function of $\beta\ell$ for a single cell 3 db coupler in Figure 4. For a 3 db coupler we have the condition:

$$\rho = \frac{Y_{1o}}{Y_{1e}} = \frac{Y_{oo}}{Y_{oe}} = \frac{B_m + B}{B} = \frac{2.414}{.414} = 5.83 \tag{6}$$

A single cell coupler is shown in Figure 5. This coupler is electrically short by a factor of two ($K = 1.414$ and $\frac{B}{Y_1} = .414$). The 71 Ω 1/8 λ high impedance coupling section is etched on either side of a .0085'' center board. With the addition of the lumped capacitors the coupler matches to 50Ω. The center board is sandwiched between two .062" dielectric boards.

The experimental data for the coupler of Figure 5 is shown in Figure 6. The coupler is undercoupled by .10 db because the centerboard was a little too thick. The thick centerboard resulted in a characteristic impedance of the coupled section that was slightly less than 71 Ω.

This type of coupler permits the construction of couplers which are shorter than conventional couplers by factors ranging from four to twelve.

Unlike the distributed coupler, the match, isolation, and phase quadrature properties of this periodically loaded hybrid are not frequency insensitive, but these errors can be held to within acceptable limits by suitably reducing the spacing between the lumped capacitors.

References

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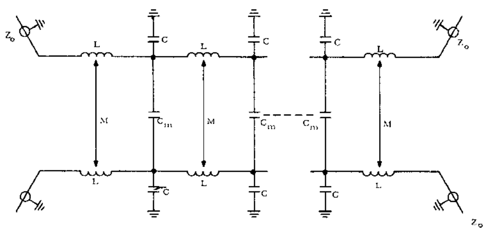

FIG. 1. Lumped Element Equivalent Circuit of a Coupled Transmission Line Directional Coupler.

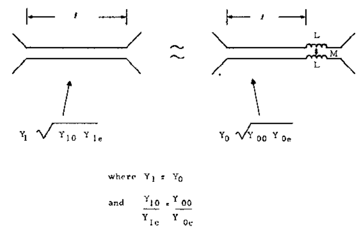

FIG. 2. Equivalence Between A Short High Impedance Coupler and a Short Low Impedance Coupler with Lumped Inductances.

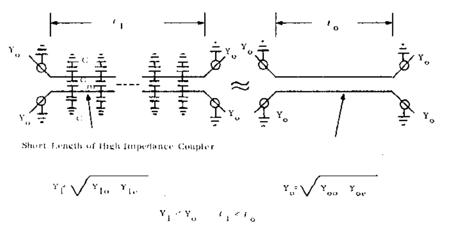

FIG. 3. Periodic Capacitive Loading of High Impedance Coupler Approximates A Long Low Impedance Coupler

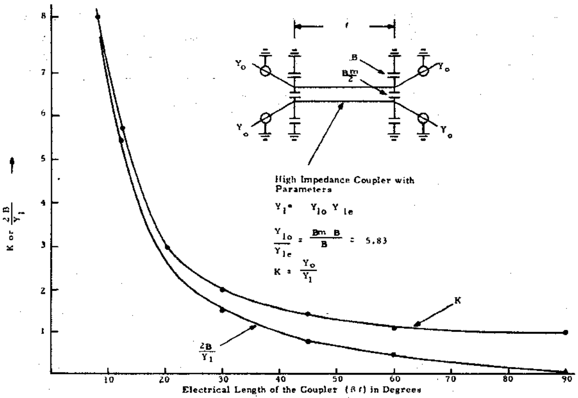

FIG. 4. Design Parameters For Single Cell 3 db Coupler

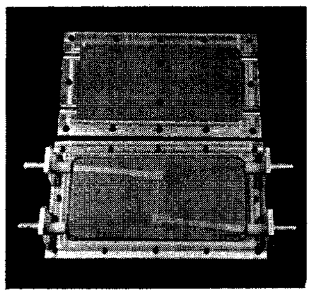

FIG. 5. Electrically Short Single Cell 3 db Coupler.

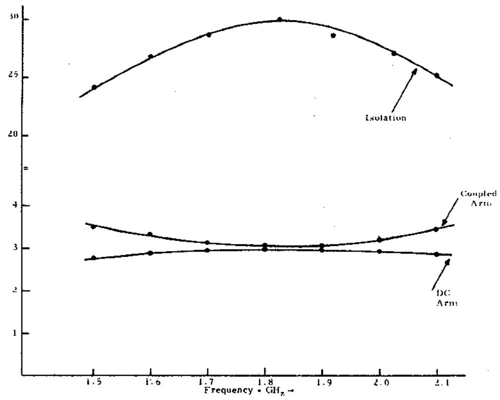

FIG. 6. 90° Coupler Electrically Short by a Factor of Two